Enrollment No: ______________________ Exam Seat No:______________________

C.U.SHAH UNIVERSITY

Summer Examination-2022

Subject Name: Advanced Real Analysis

Subject Code: 5SC03ARA1 **Branch: M.Sc. (Mathematics)**

Semester: 3 **Date: 21/04/2022** **Time: 02:30 To 05:30** **Marks: 70**

Instructions:

(1) Use of Programmable calculator and any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

SECTION – I

Q-1 **Attempt the Following questions** **(07)**

a. True/False: Each finite measure is a σ - finite measure. **(01)**

b. Suppose $(X,\mathcal{A},\mu)$ is a measure space and E is the set of σ - finite measure. Let F is a measurable subset of E then F is a set of σ - finite measure. **(02)**

c. Define: Mutually singular measure **(02)**

d. Let $(X,\mathcal{A})$ be a measurable space if $\lambda \,\&\, \mu$ are two measure on $(X,\mathcal{A})$ with $\lambda \perp \mu$ and $\lambda << \mu$ then $\lambda = 0$. **(02)**

Q-2 **Attempt all questions** **(14)**

a. Let $(X,\mathcal{A})$ be a measurable space. Suppose $f : X \to [0,\infty]$ is measurable then there exist $\{s_n\}$ of non-negative measurable simple function such that $0 \le s_1(x) \le s_2(x) \le ..... \le s_n(x) \le f(x)$ and $\lim\limits_{n} s_n(x) = f(x), x \in X$. Further if f is bounded then $s_n \to f$ is uniformly. **(08)**

b. Let $(X,\mathcal{A},\mu)$ be a measure space. Let s and t are two non-negative simple measurable function on X then prove that $\int\limits_X (s+t)\,d\mu = \int\limits_X s\,d\mu + \int\limits_X t\,d\mu$ **(06)**

OR

Q-2 **Attempt all questions** **(14)**

a. State and prove Beppo-levi's theorem. **(05)**

b. Define: Locally measurable set. Let S be a non-negative simple measurable function on a measure space $(X,\mathcal{A},\mu)$. If $\rho(E) = \int\limits_E S\,d\mu,\ \ E \in \mathcal{A}$ then ρ is a **(05)**

measure on $(X, \mathcal{A})$.

c. If $E_1, E_2 \in \mathcal{A}$ then show that $\mu(E_1 \Delta E_2) = 0 \Rightarrow \mu(E_1) = \mu(E_2)$. Moreover if μ is complete and $E_1 \in \mathcal{A}$ with $\mu(E_1 \Delta E_2) = 0$ then $E_2 \in \mathcal{A}$ **(04)**

Q-3 **Attempt all questions** **(14)**

a. State and prove Hahn-Decomposition theorem. **(07)**

b. State and prove Lusin's theorem. **(07)**

OR

Q-3 **Attempt all questions** **(14)**

a. State and prove Jordan Decomposition theorem. **(09)**

b. State and prove Lebesgue Dominated Convergence theorem. **(05)**

SECTION – II

Q-4 **Attempt the Following questions** **(07)**

a. Define: Product measure space **(01)**

b. State Tonelli's theorem. **(02)**

c. State Minkowski's inequality **(02)**

d. Suppose $f \in L^p(\mu)$ and $f = g$ a.e. and μ be a complete measure then $g \in L^p(\mu)$. **(02)**

Q-5 **Attempt all questions** **(14)**

a. State and prove Lebesgue Decomposition theorem. **(10)**

b. If ν_1 and ν_2 are finite signed measure on $(X, \mathcal{A})$ then **(04)**

i) $|\alpha \nu_1| = |\alpha||\nu_1|$ and ii) $|\nu_1 + \nu_2| \le |\nu_1| + |\nu_2|$

OR

Q-5 **Attempt all questions** **(14)**

a. State and prove Radon-Nikodym theorem. **(08)**

b. State and prove Holder's inequality. **(06)**

Q-6 **Attempt all questions** **(14)**

a. Prove that L^p spaces are complete spaces. **(08)**

b. State and prove Density theorem. **(06)**

OR

Q-6 **Attempt all Questions** **(14)**

a. State and prove Caratheodory theorem. **(08)**

b. Prove the set of all simple measurable function f vanishing outside a set of finite measure is dense in $L^p(\mu)$, where $1 \le p < \infty$. **(06)**